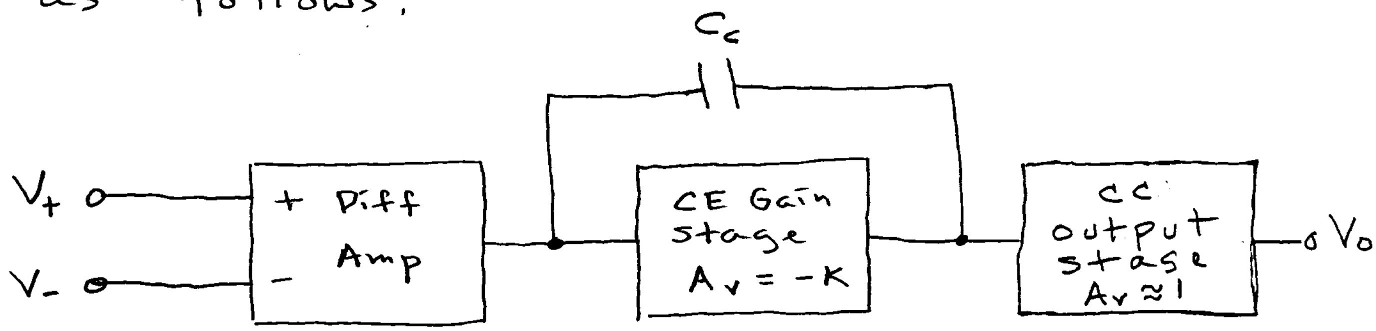
The Operational Amplifier Model

The op amp has two inputs and one output. General purpose op amps are realized as a three stage amplifier diagramed as follows:

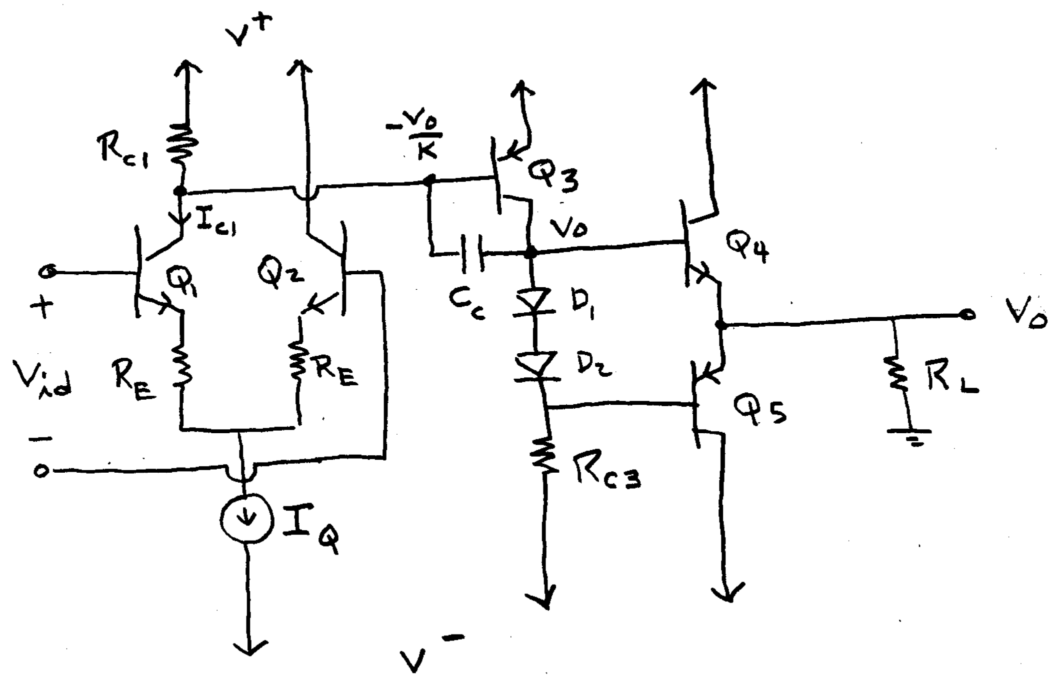


The diff amp subtracts the two input voltages to produce a difference voltage that drives a high gain stage which is usually a CE gain stage. The output stage is a complementary CC stage which supplies current gain to drive a load resistor.

The bandwidth is set by a compensating capacitor C_c which connects from the output to the input of the CE stage. At low frequencies, C_c is an open circuit. As frequency is increased, its impedance decreases and it feeds some of the output voltage from the CE stage back into its input. Because the CE stage has an inverting gain, the voltage fed back through C_c tends to cancel the input voltage to the C_c stage. Thus the gain is reduced.

The value of C_c must be large enough to prevent the circuit from oscillating, That is, it prevents the circuit from having an output signal with no input signal. A typical value in general purpose IC op amps is 10 to 50 pF.

A simple circuit for an op amp is as follows:



Q_1 and Q_2 are a differential amplifier. We assume the current I_Q divides equally between Q_1 and Q_2 so that $r_{e1} = r_{e2} = r_e = 2V_T/I_Q$. For the analysis, we assume $\alpha_1 = \alpha_2 = 1$.

Q_3 is a CE stage which we assume has a gain $-K$.

Q_4 and Q_5 are a complementary CC output stage. D_1 and D_2 bias Q_4

and Q_5 to prevent crossover distortion. We assume the gain of $Q_4 - Q_4$ is unity.

Thus if V_o is the output voltage, the voltage at the collector of Q_3 is V_o. The voltage at the base of Q_3 is $-V_o/K$.

Let $R_{eq} = R_{C1} \parallel r_{ib3}$. If r_{o1} is neglected, the node equation at the base of Q_3 is

$$I_{c1} + \left(\frac{-V_o}{K}\right)\left(\frac{1}{R_{eq}}\right) + \left(-\frac{V_o}{K} - V_o\right) C s = 0$$

$$\Rightarrow \quad V_o = \frac{I_{c1}}{\frac{1}{KR_{eq}} + \left(\frac{1}{K} + 1\right) C s}$$

$$= \frac{KR_{eq}}{1 + R_{eq}(1+K) C s} I_{c1}$$

From our diff amp analysis,

$$I_{c1} = I_{e1} = \frac{V_{id}}{2(r_e + R_E)}$$

$$\Rightarrow \quad V_o = \frac{K R_{eq}}{1 + R_{eq}(1+K)Cs} \; \frac{V_{id}}{2(r_e + R_E)}$$

It follows that the gain can be written

$$\frac{V_o}{V_{id}} = \frac{A_o}{1 + \frac{s}{\omega_o}}$$

where
$$A_o = \frac{K R_{eq}}{2(r_e + R_E)}$$

$$\omega_o = \frac{1}{R_{eq}(1+K)C}$$

Notice that the effective value of C is increased to $(1+K)C$. This is called the Miller effect.

The transfer function for V_o/V_{id} is a low-pass function. With $s = j\omega$, its magnitude is given by

$$\left|\frac{V_o}{V_i}\right| = \frac{A_o}{\sqrt{1+\left(\frac{\omega}{\omega_o}\right)^2}}$$

For $\omega \ll \omega_o$, $\left|\frac{V_o}{V_i}\right| = A_o$

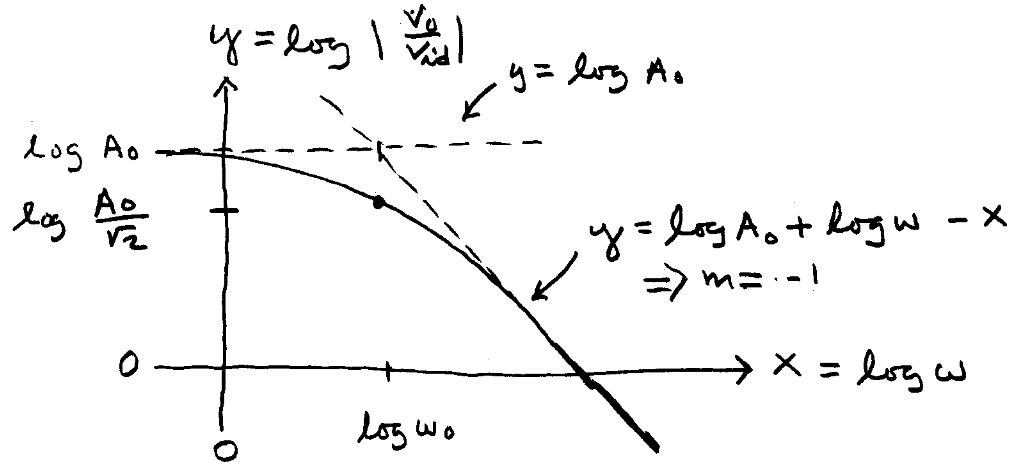
$\Rightarrow \log\left|\frac{V_o}{V_i}\right| = \log A_o$

For $\omega \gg \omega_o$, $\left|\frac{V_o}{V_i}\right| = A_o \frac{\omega_o}{\omega}$

$\Rightarrow \log\left|\frac{V_o}{V_i}\right| = \log A_o + \log \omega_o - \log \omega$

Let $y = \log\left|\frac{V_o}{V_i}\right|$ and $x = \log \omega$

We wish to plot y versus x for $\omega \ll \omega_o$ and $\omega \gg \omega_o$. Both are straight lines which intersect when $x = \log \omega_o$

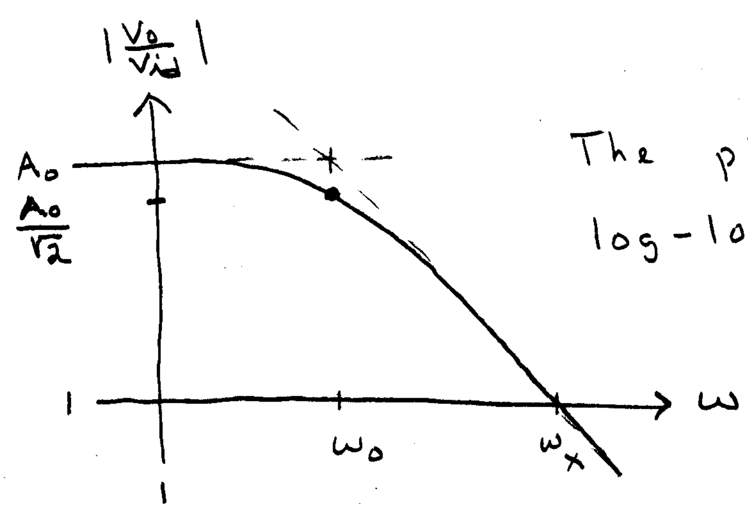


The exact curve is the solid line.

At $\omega = \omega_o$

$$\left| \frac{V_o}{V_{id}} \right| = \frac{A_o}{\sqrt{2}}$$

Next, we repeat the plot, but we label the axes without putting log before the variables



The plot uses log-log scales

The frequency at which the gain is unity is labeled ω_x. At this frequency

$$\left|\frac{V_o}{V_i}\right| = \frac{A_o}{\sqrt{1+\left(\frac{\omega_x}{\omega_o}\right)^2}} = 1$$

$$\Rightarrow \quad \omega_x = \left(\sqrt{A_o^2 - 1}\right)\omega_o$$

For $A_o^2 \gg 1$, this reduces to

$$\omega_x = A_o \omega_o$$

This is called the gain-bandwidth product. We can write

$$\omega_x = \frac{K R_{eq}}{2(r_e + R_E)} \frac{1}{R_{eq}(1+K)C}$$

$$= \frac{K}{1+K} \frac{1}{2(r_e + R_E)C}$$

For $K \gg 1$, this gives

$$\omega_x \approx \frac{1}{2(r_e + R_E)C}$$

The frequency in Hz is

$$f_x = \frac{\omega_x}{2\pi} = \frac{1}{4\pi(r_e + R_E)C}$$

The value of f_x is a basic specification for op amps. For general purpose op amps, its value is typically in the range from 1 MHz to 5 MHz. The value of A_0 is typically about 2×10^5. For these values, $f_0 = \omega_0 / 2\pi$ is in the range of 5 Hz to 25 Hz.