ECE3050 Homework Set 9

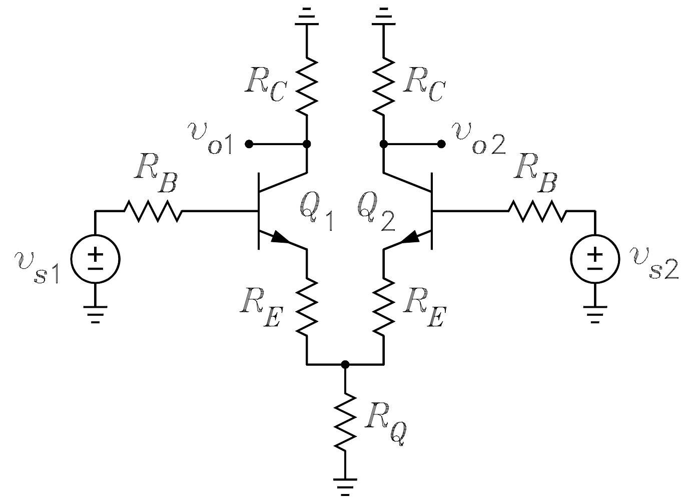
1. The figure shows the ac signal circuit of a BJT differential amplifier. It is given that $V_T = 0.025\,\text{V}$, $\beta = 99$, $r_x = 30\,\Omega$, $I_E = 1.5\,\text{mA}$, $R_B = 1.5\,\text{k}\Omega$, $R_E = 50\,\Omega$, $R_C = 12\,\text{k}\Omega$, and $R_Q = 10\,\text{k}\Omega$. To simplify the problem, assume that that $r_0 = \infty$.

   (a) Show that $r'_e = 31.967\,\Omega$.

   (b) Show that the Thévenin equivalent circuits seen looking out of the emitters in the ac signal circuit consist of the voltages and resistances $v_{te1} = 0.992v_{s2}$, $v_{te2} = 0.992v_{s1}$, and $R_{te1} = R_{te2} = 131.3\,\Omega$.

   (c) Use the emitter equivalent circuits on the simplified T model to show that $i'_{e1} = (v_{s1} - 0.992v_{s2})/163.267$ and $i'_{e2} = (v_{s2} - 0.992v_{s1})/163.267$.

   (d) Use the collector equivalent circuits of the simplified T model to show that $v_{o1} = -72.764v_{s1} + 72.173v_{s2}$ and $v_{o2} = -72.764v_{s2} + 72.173v_{s1}$.



2. An alternate solution to the differential amplifier of Problem 1 is to use common-mode and differential.

   (a) For the common-mode input signals $v_{s1} = v_{s2} = v_{s(cm)}$, use the simplified T model to show that the common-mode gain to each output is given by

$$A_{cm} = \frac{v_{o1}}{v_{s(cm)}} = \frac{v_{o2}}{v_{s(cm)}} = -0.592$$

   (b) For the differential input signals $v_{s1} = v_{s(d)}/2$ and $v_{s2} = -v_{s(d)}/2$, use the simplified T model to show that the differential gain is given by

$$A_d = \frac{v_{o1}}{v_{s(d)}} = -\frac{v_{o2}}{v_{s(d)}} = -72.468$$

   (c) Solve for the common-mode rejection ratio at the v_{o1} output to show that

$$CMRR = \frac{\left|v_{o1}/v_{s(d)}\right|}{\left|v_{o1}/v_{s(cm)}\right|} = \frac{\left|v_{o2}/v_{s(d)}\right|}{\left|v_{o2}/v_{s(cm)}\right|} = 122.501 = 41.763\,\text{dB}$$

(d) For $v_{s(cm)} = (v_{s1} + v_{s2})/2$ and $v_{s(d)} = v_{s1} - v_{s2}$, show that

$$\begin{aligned} v_{o1} &= \left(A_d + \frac{A_{cm}}{2}\right) v_{s1} - \left(A_d - \frac{A_{cm}}{2}\right) v_{s2} = -72.764 v_{s1} + 72.173 v_{s2} \\ v_{o2} &= -\left(A_d - \frac{A_{cm}}{2}\right) v_{s1} + \left(A_d + \frac{A_{cm}}{2}\right) v_{s2} = 72.173 v_{s1} - 72.764 v_{s2} \end{aligned}$$

(e) Show that

$$v_{o(d)} = v_{o1} - v_{o2} = -144.937\,(v_{s1} - v_{s2})$$

3. A differential amplifier has a differential gain of -100 and a common-mode gain of -0.01. The input voltages are given by $v_{i1} = 0.1 \sin \omega_1 t - 0.01 \sin \omega_2 t$ and $v_{i2} = 0.1 \sin \omega_1 t + 0.01 \sin \omega_2 t$.

(a) Show that the differential and common-mode input voltages are given by

$$v_{id} = -0.02 \sin \omega_2 t \qquad v_{icm} = 0.1 \sin \omega_1 t$$

(b) If the output is taken from the collector of Q_1, show that

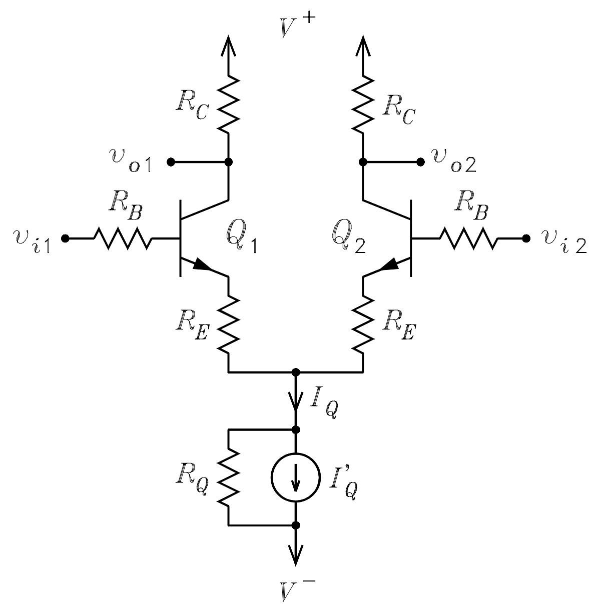
$$v_{o1} = -0.001 \sin \omega_1 t + 2 \sin \omega_2 t$$

(c) If the ω_1 term is an unwanted interference signal and the ω_2 term is a desired signal, show that the input signal-to-noise ratio is $SNR_{in} = -14\,\text{dB}$, which is pretty low. Show that the output signal-to-noise ratio is $SNR_{out} = 66\,\text{dB}$.

(d) Show that the improvement in the signal-to-noise ratio between the input and the output is 80 dB. Note: This problem illustrates how a common-mode noise voltage on a signal pair can be reduced by a large amount with a differential amplifier input stage.

(e) What is the $CMRR$? Answer: 80 dB.

4. For the diff amp shown, it is given that $I'_Q = 1.8\,\text{mA}$, $R_Q = 20\,\text{k}\Omega$, $R_C = 7500\,\Omega$, $R_E = 30\,\Omega$, $R_B = 100\,\Omega$, $r_x = 20\,\Omega$, $V^+ = +15\,\text{V}$, $V^- = -15\,\text{V}$, $V_{BE} = 0.65\,\text{V}$, $\beta = 149$, $V_A = \infty$, and $V_T = 25\,\text{mV}$.



(a) With $v_{i1} = v_{i2} = 0$, show that $I_{E1} = I_{E2} = 1.258\,\text{mA}$, $r_0 = \infty$, $r_e = 19.876\,\Omega$, $r'_e = 21.076\,\Omega$, and $r_{ib} = 15.08\,\text{k}\Omega$.

(b) Show that $R_{te} = 80.946\,\Omega$ for each BJT, $v_{te1} = 0.997 v_{i2}$, and $v_{te2} = 0.997 v_{i1}$.

(c) Show that $i'_c/v_{tb} = -i'_c/v_{te} = 9.704 \times 10^{-3}\,\Omega$ for each BJT.

(d) Show that $v_{o1}/v_{i1} = v_{o2}/v_{i2} = -72.778$ and $v_{o2}/v_{i1} = v_{o1}/v_{i2} = +72.593$.

(e) If the output is taken from either the collector of Q_1 or the collector of Q_2, show that $CMRR = 392.071$ or $51.876\,\text{dB}$.

(f) For $v_{i1} = v_{i(d)}/2$ and $v_{i2} = -v_{i(d)}/2$, show that the differential input resistance between the two inputs in series is $r_{i(d)} = 10.2\,\text{k}\Omega$.

(g) For $v_{i1} = v_{i2} = v_{i(cm)}$, show that the common-mode input resistance to the two inputs in parallel is $r_{i(cm)} = 4.005\,\text{M}\Omega$.