Oscillators

Conditions for Oscillations

Figure 1 shows the block diagram of an amplifier $A(s)$ with a feedback network $b(s)$ that can be connected from its output to its input by flipping the switch from position A to position B. In position A, the switch connects the feedback network to a test signal source V_t. The transfer function from V_t to V_o is

$$\frac{V_o}{V_t} = b(s) A(s)$$

Let $s = j\omega$. Suppose there is some frequency at which $V_o/V_t = 1\angle 0\,^\circ$. If the test signal source puts out a sine wave at this frequency, the amplifier output will be a sine wave of the same amplitude and phase. If the switch is changed to position B, the signal input to the feedback network will not change. In this case the output signal from the amplifier drives its input and the circuit is a stable oscillator.

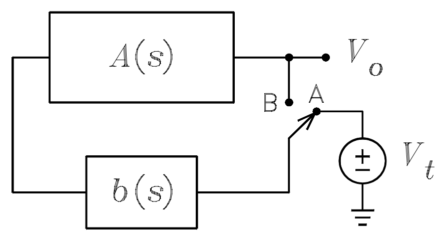

Figure 1: Block diagram of an oscillator.

If $|V_o/V_t| > 1$ at the frequency for which $\angle V_o/V_i = 0\,^\circ$, the circuit will oscillate when the switch is closed and the amplitude of the output voltage will increase with time until the amplifier overloads or clips. If $|V_o/V_t| < 1$ at the frequency for which $\angle V_o/V_i = 0\,^\circ$, the amplitude of the output voltage will decrease or damp out with time until the output becomes zero. Thus the condition for a stable output sine wave is the loop gain $|V_o/V_t|$ must be unity at the frequency for which its phase is $0\,^\circ$. A means for varying the gain so as to maintain a unity gain is usually incorporated into practical oscillator designs.

A classic example is the vacuum tube Wien Bridge Oscillator that was the founding product of Hewlett-Packard Corp. A simplified version of the circuit is shown in Figure 2. The designers used a small light bulb shown in the figure to regulate the gain. The temperature coefficient of the filament in the bulb caused its resistance to change when the current through it changed. This resistance change varied the gain of the amplifier to maintain a loop gain of unity.

Wien Bridge Oscillator

Figure 3 shows an op-amp Wien Bridge Oscillator circuit. It is based on a network originally developed by Max Wien in 1891. In order to calculate the loop-gain transfer function, the input to the feedback network is broken at the two slant lines. For an ideal op amp, $A(s)$ is a constant and is given by

$$A(s) = \frac{V_o}{V_+} = 1 + \frac{R_F}{R_3} = K$$

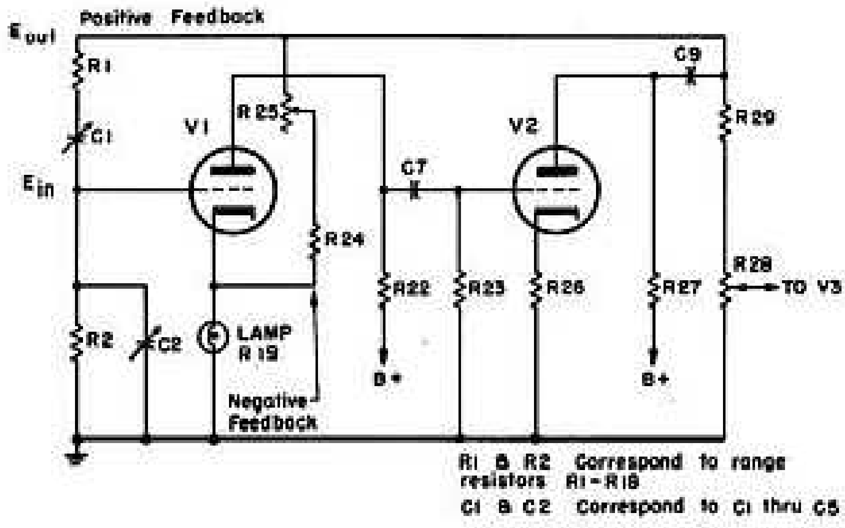

Figure 2: Simplified circuit diagram of the Hewlett-Packard vacuum tube oscillator.

The transfer function $b(s)$ for the feedback network is

$$b(s) = \frac{V_+}{V_o'} = \frac{R_1 \| \dfrac{1}{C_1 s}}{R_2 + \dfrac{1}{C_2 s} + R_1 \| \dfrac{1}{C_1 s}} = \frac{\dfrac{R_1}{1 + R_1 C_1 s}}{\dfrac{1 + R_2 C_2 s}{C_2 s} + \dfrac{R_1}{1 + R_1 C_1 s}}$$

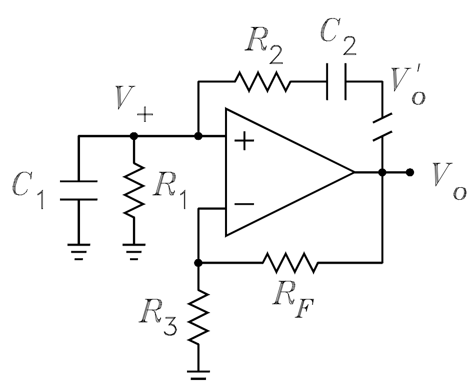

Figure 3: Op-amp Wien Bridge Oscillator.

Simplification yields

$$b(s) = \frac{R_1 C_2 s}{R_1 R_2 C_1 C_2 s^2 + (R_1 C_1 + R_2 C_2 + R_1 C_2) s + 1}$$

Thus the overall loop-gain transfer function is

$$\frac{V_o}{V_o'} = K b(s) = K \frac{R_1 C_2 s}{R_1 R_2 C_1 C_2 s^2 + (R_1 C_1 + R_2 C_2 + R_1 C_2) s + 1}$$

For $s = j\omega$, this becomes

$$\frac{V_o}{V_o'} = K \frac{j\omega R_1 C_2}{1 - \omega^2 R_1 R_2 C_1 C_2 + j\omega \left(R_1 C_1 + R_2 C_2 + R_1 C_2\right)}$$

It can be seen that the angle of this transfer function is 0° if $1 - \omega^2 R_1 R_2 C_1 C_2 = 0$, or equivalently

$$\omega = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}$$

At this frequency, the transfer function is given by

$$\frac{V_o}{V_o'} = K \frac{R_1 C_2}{R_1 C_1 + R_2 C_2 + R_1 C_2}$$

If this is equal to unity, the circuit will oscillate with a stable output when the loop is closed.

The circuit is often designed with $R_1 = R_2 = R$ and $C_1 = C_2 = C$. In this case, the frequency of oscillation is given by

$$\omega = \frac{1}{RC}$$

and the loop-gain at this frequency is

$$\frac{V_o}{V_o'} = \frac{K}{3}$$

It follows that $K = 3$ is the condition for stable oscillations at this frequency. This requires the condition $R_F = 2R_3$. A method that is often used to regulate the gain for stable oscillations is to connect a resistor in series with a JFET in parallel with R_3. The JFET is operated in the triode or linear mode. A diode detector is used to generate a dc voltage that varies with the amplitude of the output oscillations. This dc voltage is applied to the gate of the JFET to vary its resistance. The varying resistance controls the gain of the circuit.

Wikipedia Reference: http://en.wikipedia.org/wiki/Wien_bridge_oscillator

Phase Shift Oscillator

The phase shift oscillator makes use of an amplifier with an inverting gain, i.e. its gain is negative. A negative gain is equivalent to a phase shift of $\pm 180^\circ$. For the circuit to be an oscillator, the feedback network must introduce an additional phase of $\pm 180^\circ$ so that the total phase is 0° (or $\pm 360^\circ$). A phase shift of $+180^\circ$ can be obtained by cascading two RC high-pass filters. However, at the frequency where the phase shift is exactly $+180^\circ$, the gain of the filters is zero. By cascading three RC high-pass filters, there is a frequency at which the phase shift is $+180^\circ$ and the gain is non-zero. Such a circuit is shown in Figure 4.

The figure shows the feedback loop broken so that the loop transfer function can be written. It is given by

$$\frac{V_o}{V_o'} = \frac{V_o}{V_b} \times \frac{V_b}{V_a} \times \frac{V_a}{V_o'}$$

where

$$\frac{V_o}{V_b} = -\frac{R_F}{\dfrac{1}{Cs} + R} = -\frac{R_F}{R} \frac{RCs}{1 + RCs}$$

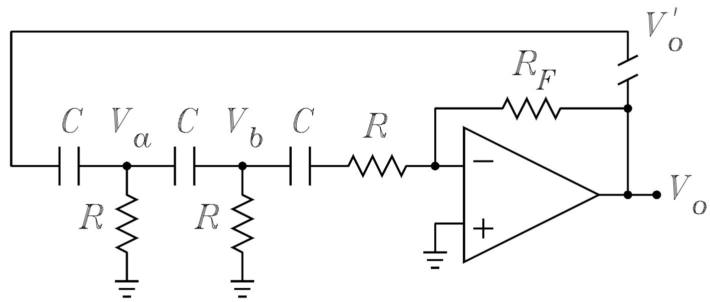

Figure 4: Phase-shift oscillator.

Admittance voltage division can be used to solve for V_b/V_a and V_a/V_o'. These are given by

$$\begin{aligned}\frac{V_b}{V_a} &= \frac{Cs}{Cs+\dfrac{1}{R}+\dfrac{1}{R+\dfrac{1}{Cs}}} \\ &= \frac{RCs\,(1+RCs)}{1+3RCs+(RCs)^2}\end{aligned}$$

$$\begin{aligned}\frac{V_a}{V_o'} &= \frac{Cs}{Cs+\dfrac{1}{R}+\dfrac{1}{\dfrac{1}{Cs}+\dfrac{1}{\dfrac{1}{R}+\dfrac{1}{\dfrac{1}{Cs}+R}}}} \\ &= \frac{RCs\left[1+3RCs+(RCs)^2\right]}{1+5RCs+6\,(RCs)^2+(RCs)^3}\end{aligned}$$

It follows that V_o/V_o' is given by

$$\begin{aligned}\frac{V_o}{V_o'} &= \frac{V_o}{V_b}\times\frac{V_b}{V_a}\times\frac{V_a}{V_o'} \\ &= -\frac{R_F}{R}\frac{RCs}{1+RCs}\frac{RCs\,(1+RCs)}{1+3RCs+(RCs)^2}\frac{RCs\left[1+3RCs+(RCs)^2\right]}{1+5RCs+6\,(RCs)^2+(RCs)^3} \\ &= -\frac{R_F}{R}\frac{(RCs)^3}{1+5RCs+6\,(RCs)^2+(RCs)^3}\end{aligned}$$

For $s=j\omega$, the numerator is a negative imaginary number for all frequencies. For the loop-gain to be $1\angle 0^\circ$ at any frequency, the denominator must also be a negative imaginary number. This occurs for the angular frequency ω_0 given by

$$\omega_0 = \frac{1}{\sqrt{6}RC}$$

At this frequency, the transfer function reduces to

$$\frac{V_o}{V_o'} = -\frac{R_F}{R}\frac{-j\left(1/\sqrt{6}\right)^3}{1+j5\left(1/\sqrt{6}\right)-1-j\left(1/\sqrt{6}\right)^3} = \frac{R_F}{R}\frac{1/6}{5-1/6} = \frac{R_F}{R}\frac{1}{29}$$

For the loop-gain to be unity, R_F must have the value

$$R_F = 29R$$

For this value, the circuit will oscillate at the frequency

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{6}RC}$$