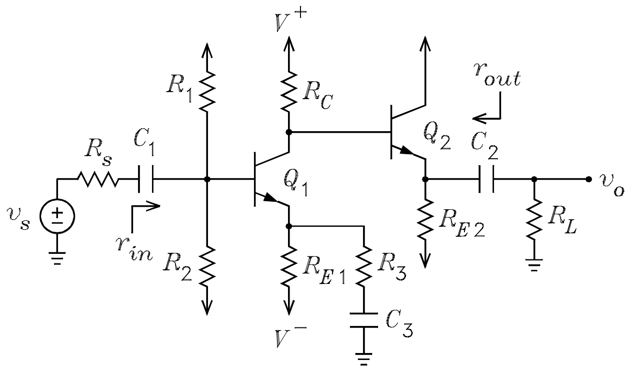
CE - CC Amplifier Example

For the circuits in the figure, it is given that $V^+ = 10\,\text{V}$, $V^- = -10\,\text{V}$, $R_s = 5\,\text{k}\Omega$, $R_1 = 100\,\text{k}\Omega$, $R_2 = 120\,\text{k}\Omega$, $R_{E1} = 2\,\text{k}\Omega$, $R_3 = 51\,\Omega$, $R_C = 2.4\,\text{k}\Omega$, $R_{E2} = 2\,\text{k}\Omega$, $R_L = 1\,\text{k}\Omega$, $V_{BE} = 0.65\,\text{V}$, $V_T = 0.025\,\text{V}$, $\alpha = 0.99$, $\beta = 99$, $r_x = 20\,\Omega$, and $r_0 = 50\,\text{k}\Omega$. The capacitors are ac short circuits and dc open circuits.



DC Solution

The dc solution for Q_1 is the same as for the CE amplifier and is repeated. To solve for I_{E1}, replace the capacitors with open circuits. Look out the base and form a Thévenin equivalent circuit. We have

$$\begin{aligned}
V_{BB1} &= \frac{V^+R_2 + V^-R_1}{R_1 + R_2} = 10\frac{120\,\text{k}\Omega}{100\,\text{k}\Omega + 120\,\text{k}\Omega} - 10\frac{100\,\text{k}\Omega}{100\,\text{k}\Omega + 120\,\text{k}\Omega} = \frac{10}{11} \\
R_{BB1} &= R_1 \| R_2 = 100\,\text{k}\Omega \| 120\,\text{k}\Omega = 54.55\,\text{k}\Omega \\
V_{EE1} &= V^- = -10 \\
R_{EE1} &= R_E = 2\,\text{k}\Omega
\end{aligned}$$

The emitter current in Q_1 is given by

$$I_{E1} = \frac{V_{BB1} - V_{BE1} - V_{EE1}}{R_{BB1}/(1+\beta) + R_{EE1}} = \frac{10/11 - 0.65 - (-10)}{54.55\,\text{k}\Omega/(1+99) + 2\,\text{k}\Omega} = 4.031\,\text{mA}$$

The ac emitter intrinsic resistance of Q_1 is

$$r_{e1} = \frac{V_T}{I_{E1}} = \frac{25\,\text{mV}}{4.031\,\text{mA}} = 6.202\,\Omega$$

Look out of the base and emitter of Q_2 and form Thévenin equivalent circuits. We have

$$\begin{aligned}
V_{BB2} &= V^+ - \alpha I_{E1} R_{C1} = 10 - 0.99 \times 4.031\,\text{mA} \times 2.4\,\text{k}\Omega = 0.4223\,\text{V} \\
R_{BB2} &= 2.4\,\text{k}\Omega \\
V_{EE2} &= V^- = -10\,\text{V} \\
R_{EE2} &= R_{E2} = 2\,\text{k}\Omega
\end{aligned}$$

The emitter current in Q_2 is given by

$$I_{E2} = \frac{V_{BB2} - V_{BE2} - V_{EE2}}{R_{BB2}/(1+\beta) + R_{EE2}} = \frac{0.4223 - 0.65 - (-10)}{2.4\,\text{k}\Omega/(1+99) + 2\,\text{k}\Omega} = 4.828\,\text{mA}$$

The ac emitter intrinsic resistance of Q_2 is

$$r_{e2} = \frac{V_T}{I_{E2}} = \frac{25\,\text{mV}}{4.828\,\text{mA}} = 5.178\,\Omega$$

AC Solution - Method 1

Zero the dc supplies and short the capacitors. Look out the base of Q_1 and make a Thévenin equivalent circuit. We have

$$\begin{aligned} v_{tb1} &= v_s \frac{R_1 \| R_2}{R_s + R_1 \| R_2} = v_s \frac{100\,\text{k}\Omega \| 120\,\text{k}\Omega}{5\,\text{k}\Omega + 100\,\text{k}\Omega \| 120\,\text{k}\Omega} = \frac{v_s}{1.092} = 0.9160 v_s \\ R_{tb1} &= R_s \| R_1 \| R_2 = 5\,\text{k}\Omega \| 100\,\text{k}\Omega \| 120\,\text{k}\Omega = 4.580\,\text{k}\Omega \end{aligned}$$

The Thévenin equivalent circuit looking into the i'_{e1} branch is v_{tb1} in series with r'_{e1}, where

$$r'_{e1} = \frac{R_{tb1} + r_{x1}}{1 + \beta_1} + r_{e1} = \frac{4.580\,\text{k}\Omega + 20}{1 + 99} + 6.202 = 52.20\,\Omega$$

The resistance looking out of the emitter of Q_1 is

$$R_{te1} = R_E \| R_3 = 2\,\text{k}\Omega \| 51 = 49.73\,\Omega$$

The resistance looking into the collector of Q_1 is

$$r_{ic1} = \frac{r_{01} + r'_{e1} \| R_{te1}}{1 - \alpha_1 R_{te1} / \left(r'_{e1} + R_{te1}\right)} = \frac{50\,\text{k}\Omega + 52.20 \| 49.73}{1 - 0.99 \times 49.73 / \left(49.73 + 52.20\right)} = 97.76\,\text{k}\Omega$$

The short circuit collector output current from Q_1 is

$$\begin{aligned} i_{c1(sc)} &= G_{mb1} v_{tb1} = \frac{\alpha_1}{r'_{e1} + R_{te1}} \frac{r_{01} - R_{te1}/\beta_1}{r_{01} + r'_{e1} \| R_{te1}} v_{tb1} \\ &= \frac{0.99}{52.20 + 49.73} \frac{50\,\text{k}\Omega - 49.73/99}{50\,\text{k}\Omega + 52.20 \| 49.73} v_{tb1} = \frac{v_{tb1}}{103.0} = \frac{v_s}{112.4} \end{aligned}$$

Look out of the base of Q_2 and make a Thévenin equivalent circuit. We have

$$\begin{aligned} v_{tb2} &= -i_{c(\text{sc})} R_C \| r_{ic1} = \frac{-v_s}{112.4} \times 2.4\,\text{k}\Omega \| 97.76\,\text{k}\Omega = -20.84 v_s \\ R_{tb2} &= R_C \| r_{ic1} = 2.4\,\text{k}\Omega \| 97.76\,\text{k}\Omega = 2.342\,\text{k}\Omega \end{aligned}$$

Replace Q_2 with its simplified T model. Looking into the r'_{e2} branch, we see v_{tb2} in series with r'_{e2} given by

$$r'_{e2} = \frac{R_{tb2} + r_{x2}}{1 + \beta_2} + r_{e2} = \frac{2.342\,\text{k}\Omega + 20}{1 + 99} + 5.178 = 28.80$$

The resistance seen looking out of the emitter of Q_2 is

$$R_{te2} = R_{E2} \| R_L = 666.7\,\Omega$$

By voltage division, v_o is given by

$$v_o = v_{tb2} \frac{r_{02} \| R_{te2}}{r'_{e2} + r_{02} \| R_{te2}} = -20.84 v_s \frac{50\,\text{k}\Omega \| 666.7}{28.80 + 50\,\text{k}\Omega \| 666.7} = -19.97 v_s$$

Thus the voltage gain is

$$\frac{v_o}{v_s} = -19.97$$

The output resistance is

$$r_{\text{out}} = R_{E2} \| r_{02} \| r'_{e2} = 2\,\text{k}\Omega \| 50\,\text{k}\Omega \| 28.80 = 28.38\ \Omega$$

To solve for the input resistance, we need r_{ib1}. To calculate this, we need R_{tc1}, which requires us to know r_{ib2}. For the latter, we have

$$\begin{aligned} r_{ib2} &= r_{x2} + (1+\beta_2)\, r_{e2} + R_{te2} \frac{(1+\beta_2)\, r_{02} + R_{tc2}}{r_{02} + R_{te2} + R_{tc2}} \\ &= 20 + (1+99)\, 5.178 + 666.7 \frac{(1+99)\, 50\,\text{k}\Omega}{50\,\text{k}\Omega + 666.7} \\ &= 66.33\,\text{k}\Omega \end{aligned}$$

Thus the resistance seen looking out of the collector of Q_1 is

$$R_{tc1} = R_C \| r_{ib2} = 2.4\,\text{k}\Omega \| 66.33\,\text{k}\Omega = 2.316\,\text{k}\Omega$$

The resistance looking into the base of Q_1 is

$$\begin{aligned} r_{ib1} &= r_{x1} + (1+\beta_1)\, r_{e1} + R_{te1} \frac{(1+\beta_1)\, r_{01} + R_{tc1}}{r_{01} + R_{te1} + R_{tc1}} \\ &= 5.391\,\text{k}\Omega \end{aligned}$$

The input resistance is

$$r_{in} = R_1 \| R_2 \| r_{ib1} = 100\,\text{k}\Omega \| 120\,\text{k}\Omega \| 5.613\,\text{k}\Omega = 5.089\,\text{k}\Omega$$

If Q_2 and R_{E2} are omitted from the circuit and the left node of C_2 is connected to the collector of Q_1, we have a common-emitter amplifier. In this case, the output voltage is

$$v_o = -i_{c1(sc)} R_C \| r_{ic1} \| R_L = \frac{-v_s}{112.4} \times 2.4\,\text{k}\Omega \| 97.76\,\text{k}\Omega \| 1\,\text{k}\Omega = -6.235 v_o$$

Thus the voltage gain drops to

$$\frac{v_o}{v_s} = -6.235$$

This is lower than with the CC stage by a factor of 3.25 or by 10.2 dB. This illustrates how a stage that has a gain less than unity can increase the gain of a circuit when it is used to drive the load resistor.

AC Solution - Method 2

For this solution, we use the r_0 approximations for Q_1. That is, we neglect the current through r_{01} in calculating $i_{c1(\text{sc})}$ but not in calculating r_{ic1}. The short circuit collector output current of Q_1 is

$$i_{c1(sc)} = G_{m1} v_{tb1} = \frac{\alpha}{r'_{e1} + R_{te1}} v_{tb1} = \frac{0.99 v_{tb1}}{52.20 + 49.73} = \frac{v_{tb1}}{103.0} = \frac{v_{s1}}{111.3}$$

Look out of the base of Q_2 and make a Thévenin equivalent circuit. We have

$$\begin{aligned} v_{tb2} &= -i_{c1(sc)} R_C \| r_{ic1} = \frac{-v_s}{111.3} \times 2.4\,\text{k}\Omega \| 97.76\,\text{k}\Omega = -21.05 v_s \\ R_{tb2} &= R_C \| r_{ic1} = 2.4\,\text{k}\Omega \| 97.76\,\text{k}\Omega = 2.342\,\text{k}\Omega \end{aligned}$$

Replace Q_2 with its simplified T model. Looking into the r'_{e2} branch, we see v_{tb2} in series with r'_{e2} given by

$$r'_{e2} = \frac{R_{tb2} + r_{x2}}{1+\beta_2} + r_{e2} = \frac{2.342\,\text{k}\Omega + 20}{1+99} + 5.178 = 28.80$$

By voltage division, v_o is given by

$$v_o = v_{tb2} \frac{r_{02} \| R_{te2}}{r'_{e2} + r_{02} \| R_{te2}} = -21.05 v_s \frac{50\,\mathrm{k}\Omega \| 666.7}{28.80 + 50\,\mathrm{k}\Omega \| 666.7} = -20.17 v_s$$

Thus the voltage gain is

$$\frac{v_o}{v_s} = -20.17$$

This differs from the answer by Method 1 by 0.99%.

The solutions for r_{out} and r_{in} are the same as for Method 1.