ECE 6416 Assignment 1

The answers to the problems are given. You must show how you get the answers for credit. Please follow the homework guidelines described at

http://users.ece.gatech.edu/~mleach/ece3050/hwguide.html

1. Let $v_n(t)$ be a random voltage that has a normal or Gaussian probability density function with a mean value of zero and a standard deviation or mean-square of v_{rms}. In a 24 h period, the total time that the waveform exceeds a value v_1 can be estimated as 24 h multiplied by the probability that $|v_n(t)| > v_1$. Calculate the time that (a) $|v_n(t)| > 3v_{rms}$ [about 4 min], (b) $|v_n(t)| > 4v_{rms}$ [about 5 s], and (c) $|v_n(t)| > 6v_{rms}$ [about 0.2 ms].

2. Two resistors R_1 and R_2 are connected in parallel. The two resistors are in thermal equilibrium.

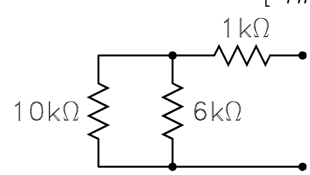
   (a) Suppose that only R_1 generates thermal noise and R_2 is noiseless, show that the average thermal noise power in watts delivered by R_1 to R_2 in the band Δf is given by

   $$P_{12} = \frac{4kTR_1R_2\Delta f}{(R_1 + R_2)^2}$$

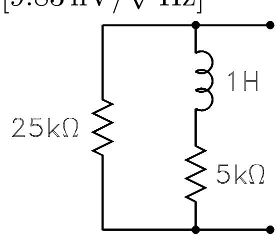
   (b) Suppose that only R_2 generates thermal noise and R_1 is noiseless, show that the average thermal noise power P_{21} delivered by R_2 to R_1 in the band Δf is given by the same expression obtained above.

   (c) Note that $P_{12} = P_{21}$. If the two answers were not the same, could the two resistors be in thermal equilibrium? How would the temperatures of the individual resistors vary with time if $P_{12} > P_{21}$?

3. Calculate the thermal spot noise voltage in $\mathrm{V}/\sqrt{\mathrm{Hz}}$ (volts per root hertz) at the standard temperature across the terminals of the circuit [$v_{rms} = 8.72\,\mathrm{nV}/\sqrt{\mathrm{Hz}}$]



4. Calculate the spot noise voltage at the output of the circuit at the frequency $f = 1.5\,\mathrm{kHz}$. Assume $T = T_0 = 290\,\mathrm{K}$. [$9.83\,\mathrm{nV}/\sqrt{\mathrm{Hz}}$]



5. A $1\,\mathrm{M}\Omega$ resistor has a dc voltage across it of $4\,\mathrm{V}$. At the frequency $f = 100\,\mathrm{Hz}$, the spot noise voltage across the resistor is $v_n/\sqrt{\Delta f} = 400\,\mathrm{nV}/\sqrt{\mathrm{Hz}}$.

   (a) Show the flicker noise coefficient is $K_f = 9 \times 10^{-13}$.

   (b) Show that the noise index is $NI = 3.17\,\mathrm{dB}$.

   (c) The mean-square short-circuit noise current generated by the resistor is given by

$$i_n^2 = \frac{4kT\Delta f}{R} + \frac{K_f I_{DC}^2 \Delta f}{f}$$

   Show that the flicker noise corner frequency is $f_{flk} = 900\,\mathrm{Hz}$.

6. A $100\,\mathrm{mH}$ lossy inductor has a measured impedance magnitude of $8\,\mathrm{k}\Omega$ at the frequency $f = 10\,\mathrm{kHz}$. Show that the open-circuit thermal spot noise voltage generated by the inductor at $10\,\mathrm{kHz}$ is $v_t/\sqrt{\Delta f} = 8.9\,\mathrm{nV}/\sqrt{\mathrm{Hz}}$. Note that $|Z|^2 = R^2 + (\omega L)^2$ for the inductor.

7. If the diode generates only shot noise and the resistor generates only thermal noise, solve for the ac rms noise output voltage over the band from $1\,\mathrm{kHz}$ to $3.5\,\mathrm{kHz}$. The diode is modeled as a shot noise current source in parallel with the diode small-signal resistance given by $r_d = \eta V_T / I_D$, where η is the emission coefficient or idealty factor and I_D is the dc current in the diode. Assume $\eta = 2$ and $V_T = 25\,\mathrm{mV}$. [$v_{rms} = 23.9\,\mathrm{nV}$]