ECE6604
PERSONAL & MOBILE COMMUNICATIONS

Week 2

Link budget, Interference and Shadow Margins, Handoff Gain, Coverage, Capacity

Reading: Chapter 1, 1.5–1.7

Receiver Sensitivity

- Receiver sensitivity refers to the ability of the receiver to detect radio signals. We would like our radio receivers to be as sensitive as possible.

- Radio receivers must detect radio waves in the presence of noise and interference.

  - External noise sources include atmospheric noise (e.g, lightning strikes), galactic noise, man made noise (e.g, automobile ignition noise), co-channel and adjacent channel interference.

  - Internal noise sources include thermal noise.

- The ratio of the desired signal power to thermal noise power before detection is commonly called the **carrier-to-noise ratio**, Γ.

- The parameter Γ is a function of the communication link parameters including transmitted power (or effective isotropic radiated power (EIRP)), path loss, receiver antenna gain, and the effective input-noise temperature of the receiving system.

- The formula that relates the link parameters to Γ is called the **link budget**.

Link Budget

- The link budget can be expressed in terms of the following parameters:

$$
\begin{aligned}
\Omega_t &= \text{transmitted carrier power} \\
G_\mathsf{T} &= \text{transmitter antenna gain} \\
L_p &= \text{path loss} \\
G_\mathsf{R} &= \text{receiver antenna gain} \\
\Omega_p &= \text{received signal power} \\
E_s &= \text{received energy per modulated symbol} \\
T_o &= \text{receiving system noise temperature in degrees Kelvin} \\
B_w &= \text{receiver noise equivalent bandwidth} \\
N_o &= \text{white noise power spectral density} \\
R_c &= \text{modulated symbol rate} \\
k &= 1.38 \times 10^{-23} = \text{Boltzmann's constant} \\
F &= \text{noise figure, typically about 3 dB} \\
L_{\mathsf{Rx}} &= \text{receiver implementation losses} \\
L_\mathsf{I} &= \text{losses due to system load (interference)} \\
M_\mathsf{shad} &= \text{shadow margin} \\
G_\mathsf{HO} &= \text{handoff gain} \\
\Omega_\mathsf{th} &= \text{receiver sensitivity}
\end{aligned}
$$

Noise Equivalent Bandwidth, B_w

- Consider an arbitrary filter with transfer function $H(f)$.

- If the input to the filter is a white noise process with power spectral density $N_o/2$ watts/Hz, then the noise power at the output of the filter is

$$
\begin{aligned}
N_{\text{out}} &= \frac{N_o}{2} \int_{-\infty}^{\infty} |H(f)|^2 df \\
&= N_o \int_{0}^{\infty} |H(f)|^2 df
\end{aligned}
$$

- Next suppose that the same white noise process is applied to an ideal low-pass filter with bandwidth B_w and d.c. response $H(0)$. The noise at the output of the filter is

$$
N_{\text{out}} = N_o B_w H^2(0)
$$

- Equating the above two equations give the **noise equivalent bandwdith**

$$
B_w = \frac{\int_{0}^{\infty} |H(f)|^2 df}{H^2(0)}
$$

- The effective received carrier power is

$$\Omega_p = \frac{\Omega_t G_\mathsf{T} G_\mathsf{R}}{L_{\mathsf{R_x}} L_p} \quad .$$

- The total input noise power to the detector is

$$N = kT_o B_w F$$

- Very often the following kT_o value at room temperature of 17 ^oC (290 ^oK) is used $kT_o = -174$ dBm/Hz,

- The received carrier-to-noise ratio defines the link budget

$$\Gamma = \frac{\Omega_p}{N} = \frac{\Omega_t G_\mathsf{T} G_\mathsf{R}}{kT_o B_w F L_{\mathsf{R_x}} L_p} \quad .$$

- The carrier-to-noise ratio, Γ, and modulated **symbol energy-to-noise ratio**, E_s/N_o, are related as follows

$$\frac{E_s}{N_o} = \Gamma \times \frac{B_w}{R_c} \quad .$$

- Hence, we can rewrite the link budget as

$$\frac{E_s}{N_o} = \frac{\Omega_t G_\mathsf{T} G_\mathsf{R}}{kT_o R_c F L_{\mathsf{R_x}} L_p} \quad .$$

- Converting into decibel units gives

$$E_s/N_{o\text{(dB)}} = \Omega_{t\ \text{(dBm)}} + G_{T\ \text{(dB)}} + G_{R\ \text{(dB)}} \tag{1}$$
$$-kT_{o\text{(dBm)/Hz}} - R_{c\ \text{(dB Hz)}} - F_{\text{(dB)}} - L_{R_x\ \text{(dB)}} - L_{p\ \text{(dB)}} \ .$$

- The **receiver sensitivity** is defined as

$$\Omega_{\text{th}} = L_{R_x} k T_o F (E_s/N_o) R_c$$

  or converting to decibel units

$$\Omega_{\text{th (dBm)}} = L_{R_x\ \text{(dB)}} + kT_{o\text{(dBm/Hz)}} + F_{\text{(dB)}} + E_s/N_{o\text{(dB)}} + R_{c\ \text{(dB Hz)}} \ .$$

- All parameters are usually fixed except for E_s/N_o. The receiver sensitivity (in dBm) is determined by the minimum acceptable E_s/N_o.

- Substituting the determined receiver sensitivity $\Omega_{\text{th (dBm)}}$ into (1) and solving for $L_{p\ \text{(dB)}}$ gives the **maximum allowable path loss**

$$L_{\text{max (dB)}} = \Omega_{t\ \text{(dBm)}} + G_{T\ \text{(dB)}} + G_{R\ \text{(dB)}} - \Omega_{\text{th (dBm)}} \ .$$

Interference Margin

- As the subscriber load increases, additional interference is generated from both inside and outside of a cell. With increased interference, the coverage area shrinks and some calls are dropped. As calls are dropped, the interference decreases and the coverage area expands.

  - the expansion/contraction of the coverage area is a phenomenon known as **cell breathing**.

- We must introduce an interference degradation margin into the link budget to account for cell breathing.

  - The received carrier-to-interference-plus-noise ratio is

  $$\Gamma_{IN} = \frac{\Omega_p}{I+N} = \frac{\Omega_p/N}{1+I/N} \ ,$$

  where I is the total interference power.

  - The net effect of such interference is to reduce the carrier-to-noise ratio Ω_p/N by the factor $L_I = (1+I/N)$. To allow for system loading, we must reduce the maximum allowable path loss by an amount equal to $L_{I \ (\text{dB})}$, otherwise known as the **interference margin**.

  - The appropriate value of $(L_I)_{\text{dB}}$ depends on the particular cellular system being deployed and the maximum expected traffic loading.

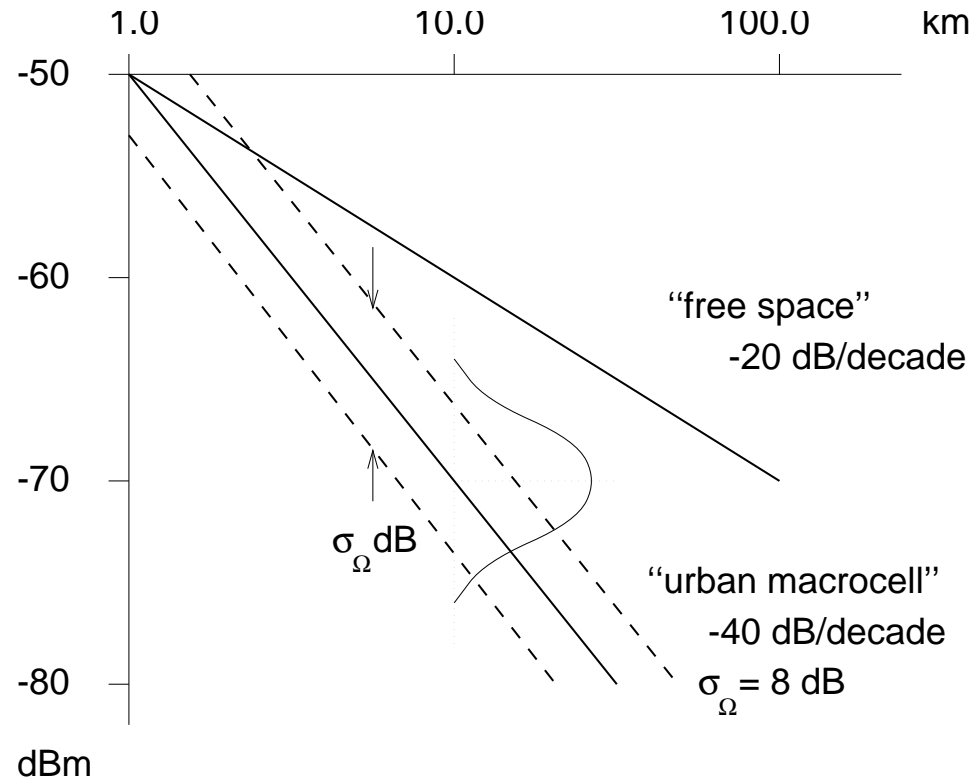# Shadowing

- With shadowing the received signal power is

$$
\begin{aligned}
\Omega_{p\ (\text{dBm})}(d) &= \mu_{\Omega_{p\ (\text{dBm})}}(d_o) - 10\beta \log_{10}(d/d_o) + \epsilon_{(\text{dB})} \\
&= \mu_{\Omega_{p\ (\text{dBm})}}(d) + \epsilon_{(\text{dB})} \;,
\end{aligned}
$$

  where the parameter $\epsilon_{(\text{dB})}$ is the error between the predicted and actual path loss.

- Very often $\epsilon_{(\text{dB})}$ is modeled as a zero-mean Gaussian or normal random variable with variance σ_Ω^2, where σ_Ω in decibels (dB) is called the **shadow standard deviation**.

- The probability density function of $\Omega_{p\ (\text{dBm})}(d)$ has the normal distribution

$$
p_{\Omega_{p\ (\text{dBm})}(d)}(x) = \frac{1}{\sqrt{2\pi}\sigma_\Omega} \exp\left\{ -\frac{\left(x - \mu_{\Omega_{p\ (\text{dBm})}}(d)\right)^2}{2\sigma_\Omega^2} \right\} \;.
$$

- Typically, σ_Ω ranges from 4 to 12 dB depending on the local topography; $\sigma_\Omega = 8$ dB is a very commonly used value.

| 1.0 | 10.0 | 100.0 | km |

-50

-60

"free space"
-20 dB/decade

-70

σ_Ω dB

"urban macrocell"
-40 dB/decade

-80

σ_Ω = 8 dB

dBm

Path loss and shadowing in a typical cellular environment.

Noise Outage

- The quality of a radio link is acceptable only when the received signal power $\Omega_{p \text{ (dBm)}}$ is greater than the receiver sensitivity $\Omega_{\text{th (dBm)}}$.

- An **outage** occurs whenever $\Omega_{p \text{ (dBm)}} < \Omega_{\text{th (dBm)}}$.

- The **edge outage probability**, P_E, is defined as the probability that $\Omega_{p \text{ (dBm)}} < \Omega_{\text{th (dBm)}}$ at the cell edge.

- The **area outage probability**, P_O, is defined as the probability that $\Omega_{p \text{ (dBm)}} < \Omega_{\text{th (dBm)}}$ when averaged over the entire cell area.

- To maintain an acceptable outage probability in the presence of shadowing, we must introduce a **shadow margin**.

Area = 0.1

$\sigma_\Omega = 8$

Ω_{th}

M_{shad}

received carrier power (dBm)
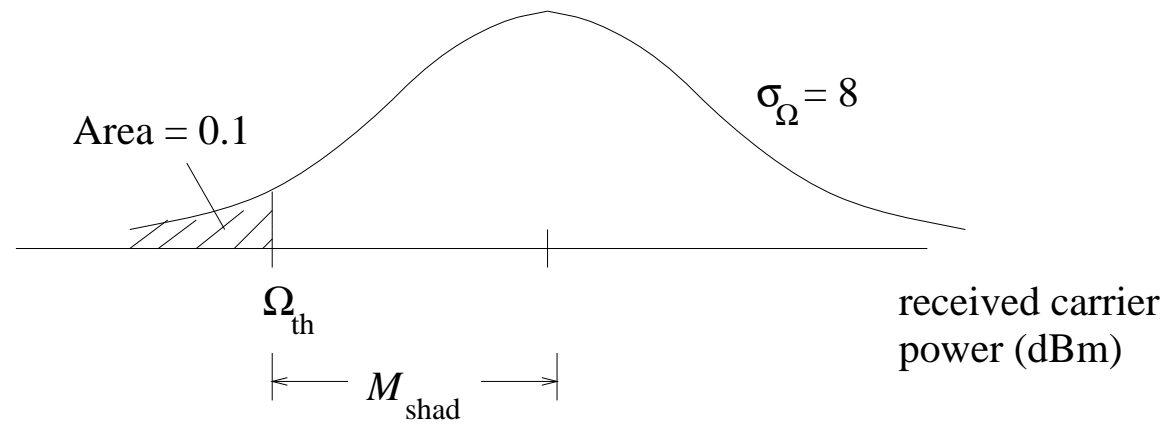
Determining the required shadow margin to give $P_E = 0.1$.

- Choose M_{shad} so that the shaded area under the Gaussian density function is equal to 0.1. Hence, we solve

$$0.1 = Q\left(\frac{M_{\text{shad}}}{\sigma_\Omega}\right) \qquad Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy$$

- We have

$$\frac{M_{\text{shad}}}{\sigma_\Omega} = Q^{-1}(0.1) = 1.28$$

- For $\sigma_\Omega = 8$ dB we have

$$M_{\text{shad}} = 1.28 \times 8 = 10.24 \text{ dB}$$

- The area outage probability (uniform user density, d^β path loss, no power control) is

$$P_O = Q(X) - \exp\left\{XY + Y^2/2\right\} Q(X + Y)$$

  where

$$X = \frac{M_{\text{shad}}}{\sigma_\Omega}, \qquad Y = \frac{2\sigma_\Omega \ln 10}{10\beta}$$

  From this we can solve for the required shadow margin, M_{shad}.

- Note that $P_O < P_E$ for the same value of M_{shad}.

Handoff Gain

- At the boundary area between two cells, we obtain a **macrodiversity** effect.

- Although the link to the serving base station may be shadowed such that $\Omega_{p \text{ (dBm)}}$ is below the receiver threshold, the link to another base station may provide a $\Omega_{p \text{ (dBm)}}$ above the receiver threshold.

- Handoffs take advantage of macrodiversity to reduce the required shadow margin over the single cell case, by an amount equal to the **handoff gain**, G_{HO}.

- There are a variety of handoff algorithms used in cellular systems. CDMA systems use **soft handoffs**, while TDMA systems usually use **hard handoffs**.

- The maximum allowable path loss with the inclusion of the margins for shadowing and interference loading, and handoff gain is

$$L_{\text{max (dB)}} = \Omega_{t \text{ (dBm)}} + G_{\text{T (dB)}} + G_{\text{R (dB)}} - \Omega_{\text{th (dBm)}} - M_{\text{shad (dB)}} - L_{I \text{ (dB)}} + G_{\text{HO (dB)}} \;.$$

Hard vs. Soft Handoff

- Consider a cluster of 7 cells; the target cell is in the center and surrounded by 6 adjacent cells. Although the MS is located in the center cell, it is possible that the MS could be connected to any one of the 7 BSs.

- We wish to the calculate the area averaged noise outage probability for the target cell, assuming that the MS location is uniform distributed over the target cell area.

- Assume that the links to the serving BS and the six neighboring BSs experience "correlated" log-normal shadowing. To generate the required shadow gains, we express the shadow gain at BS_i as

$$\epsilon_i = a\zeta + b\zeta_i \ ,$$

where

$$a^2 + b^2 = 1 \ ,$$

and ζ and ζ_i are independent Gaussian random variables with zero mean and variance σ_Ω^2.
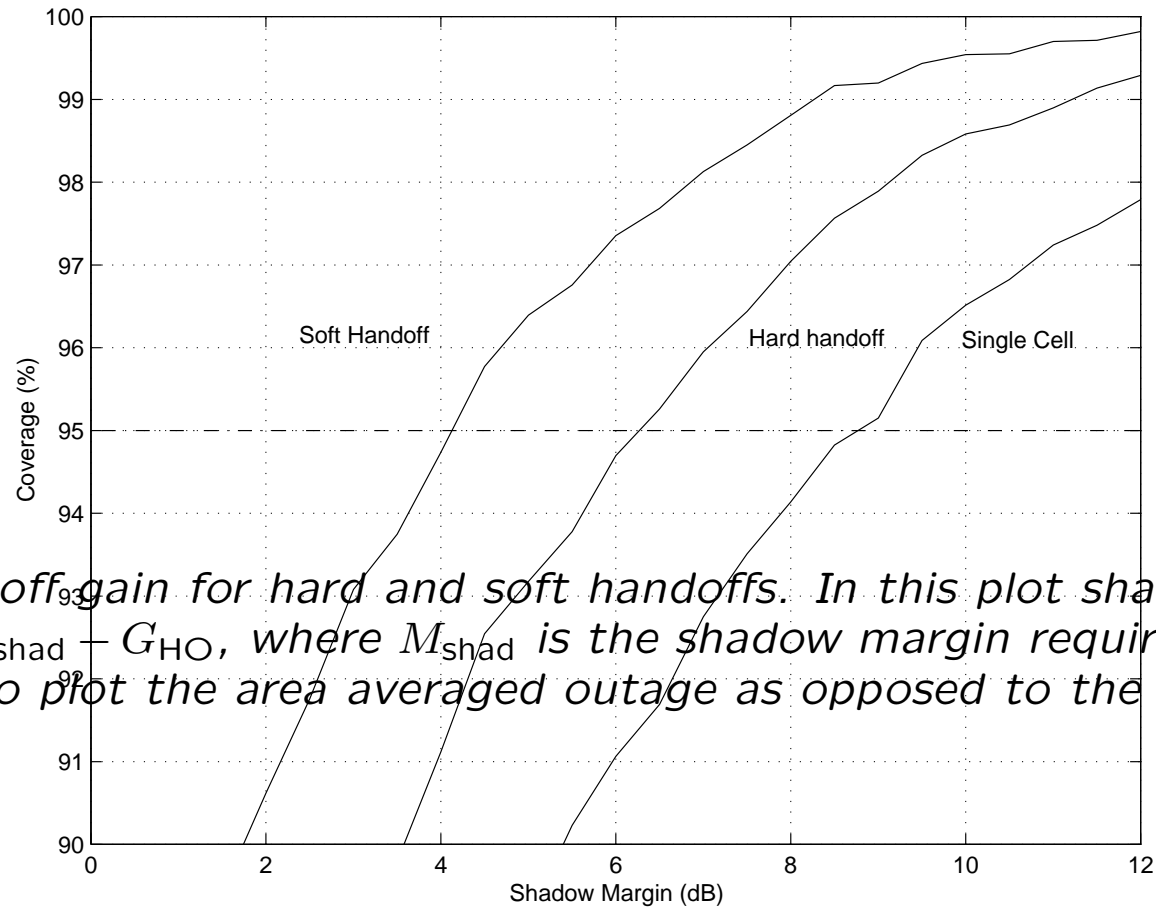
- It follows that the shadow gains (in decibel units) have the correlation

$$\mathsf{E}[\epsilon_i \epsilon_j] = a^2 \sigma_\Omega^2 = \rho \sigma_\Omega^2$$

where $\rho = a^2$ is the correlation coefficient. Here we assume that $\rho = 0.5$.

Hard vs. Soft Handoff

- "Soft handoff" algorithm: the BS that provides the largest instantaneous received signal strength is selected as the serving BS.

  - If any BS has an associated received signal power that is above the receiver sensitivity, $\Omega_{\text{th (dBm)}}$, then the link quality is acceptable; otherwise an outage will occur.

- "Hard handoff" algorithm:

  - The received signal power from the serving BS is equal to $\Omega_{p,0 \text{ (dBm)}}$. If this value exceeds the receiver sensitivity, $\Omega_{\text{th (dBm)}}$, then the link quality is acceptable.

  - Otherwise, the six surrounding BSs are evaluated for handoff candidacy by using a mobile assisted handoff algorithm. A BS that qualifies as a handoff candidate must have $\Omega_{p,k \text{ (dBm)}} - \Omega_{p,0 \text{ (dBm)}} \geq H_{\text{(dB)}}$, where $H_{\text{(dB)})}$ is the handoff hysteresis.

  - We then check those BSs passing the hysteresis test. If the received signal power for any of these BSs is above the receiver sensitivity, $\Omega_{\text{th (dBm)}}$, then link quality is acceptable; otherwise an outage occurs.

Coverage (%) axis with values 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100

Shadow Margin (dB) axis: 0, 2, 4, 6, 8, 10, 12

Labels: Soft Handoff, Hard handoff, Single Cell

Typical handoff gain for hard and soft handoffs. In this plot shadow margin is defined as $M_{\mathrm{shad}} - G_{\mathrm{HO}}$, where M_{shad} is the shadow margin required for a single cell. We also plot the area averaged outage as opposed to the edge outage.

16

Cellular Radio Coverage

- **Radio coverage** refers to the number of base stations or cell sites that are required to "cover" or provide service to a given area with an acceptable grade of service.

- The number of cell sites required to cover a given area is determined by the maximum allowable path loss and the path loss exponent.

- To compare the coverage of different cellular systems, we first determine the maximum allowable path loss, $L_{\text{max (dB)}}$, for the different systems by using a common quality of service criterion.

- Then
$$L_{\text{max (dB)}} = C + 10\beta\log_{10}d_{\text{max}}$$
where d_{max} is the radio path length that corresponds to the maximum allowable path loss and C is a constant.

- The quantity d_{max} is equal to the radius of the cell.

- To provide good coverage it is desirable that d_{max} be as large as possible.

Comparing Coverage

- Suppose that System 1 has $L_{\text{max (dB)}} = L_1$ and System 2 has $L_{\text{max (dB)}} = L_2$, with corresponding radio path lengths of d_1 and d_2, respectively. The difference in the maximum allowable path loss is related to the cell radii by

$$
\begin{aligned}
L_1 - L_2 &= 10\beta \left(\log_{10}d_1 - \log_{10}d_2\right) \\
&= 10\beta \left(\log_{10}\frac{d_1}{d_2}\right)
\end{aligned}
$$

or looking at things another way

$$
\frac{d_1}{d_2} = 10^{(L_1 - L_2)/(10\beta)}
$$

Since the area of a cell is equal to $A = \pi d^2$ (assuming a circular cell) the ratio of the cell areas is

$$
\frac{A_1}{A_2} = \frac{d_1^2}{d_2^2} = \left(\frac{d_1}{d_2}\right)^2
$$

and, hence,

$$
\frac{A_1}{A_2} = 10^{(L_1 - L_2)/(5\beta)} \quad .
$$

- Suppose that A_{tot} is the total geographical area to be covered. Then the ratio of the required number of cell sites for Systems 1 and 2 is

$$\frac{N_1}{N_2} = \frac{A_{\text{tot}}/A_1}{A_{\text{tot}}/A_2} = \frac{A_2}{A_1} = 10^{-(L_1 - L_2)/(5\beta)}$$

- **Example:** Suppose that $\beta = 3.5$ and $L_1 - L_2 = 2$ dB.

  - $N_2/N_1 = 1.30$.

  - Conclusion: System 2 requires 30% more base stations to cover the same geographical area for only a 2 dB difference in link budget.

- Note that the required interference margin and realized handoff gain can have a huge impact. Coverage comparisons should be done under conditions of equal traffic loading.

Spectral Efficiency

- Spectral efficiency can be expressed in terms of the following parameters:

$$
\begin{aligned}
G_c &= \text{offered traffic per channel (Erlangs/channel)} \\
N_{\text{slot}} &= \text{number of channels per RF carrier} \\
N_c &= \text{number of RF carriers per cell area (carriers/m}^2) \\
W_{\text{sys}} &= \text{total system bandwidth (Hz)} \\
A &= \text{area per cell (m}^2) \quad .
\end{aligned}
$$

One Erlang is the traffic intensity in a channel that is continuously occupied, so that a channel occupied for $x\%$ of the time carriers $x/100$ Erlangs. Adjustment of this parameter controls the system loading and it is important to compare systems at the same traffic load level.

- For an N-cell reuse cluster, we can define the spectral efficiency as follows:

$$
\eta_{\text{S}} = \frac{N_c N N_{\text{slot}} G_c}{W_{\text{sys}} A} \quad \text{Erlangs/m}^2\text{/Hz} \quad .
$$

Spectral Efficiency (con'td)

- Recognizing that the bandwidth per channel, W_c, is equal to $W_\text{sys}/(NN_cN_\text{slot})$, the spectral efficiency can be written as the product of three efficiencies, viz.,

$$
\begin{aligned}
\eta_\text{S} &= \frac{1}{W_c} \cdot \frac{1}{A} \cdot G_c \\
&= \eta_\text{B} \cdot \eta_\text{C} \cdot \eta_\text{T} \ ,
\end{aligned}
$$

  where

$$
\begin{aligned}
\eta_\text{B} &= \text{bandwidth efficiency} \\
\eta_\text{C} &= \text{spatial efficiency} \\
\eta_\text{T} &= \text{trunking efficiency}
\end{aligned}
$$

- Unfortunately, these efficiencies are not independent so the optimization of spectral efficiency is quite complicated.

- For cellular systems, the number of channels per cell (or cell sector) is sometimes used instead of the Erlang capacity. We have

$$
N_cN_\text{slot} = \frac{W_\text{sys}}{W_c \cdot N}
$$

  where, again, W_c is the bandwidth per channel and N_slot is the number of traffic channels multiplexed on each RF carrier.
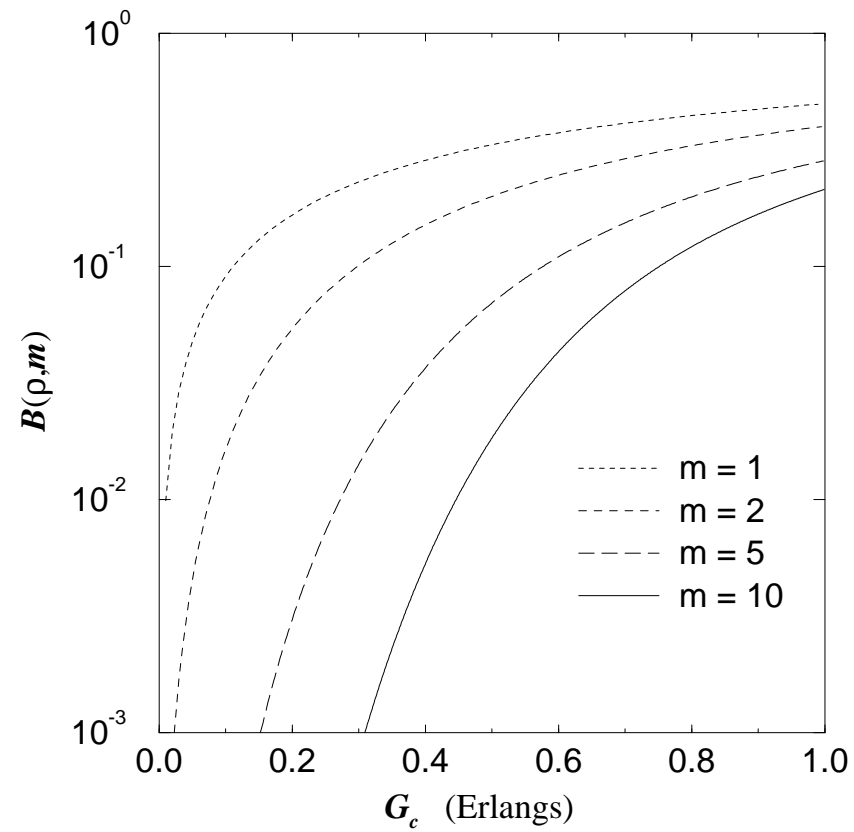
Trunking Efficiency

- The **cell Erlang capacity** equal to the traffic carrying capacity of a cell (in Erlangs) for a specified call blocking probability.

- The Erlang capacity can be calculated using the famous **Erlang-B formula**

$$B(\rho, m) = \frac{\rho^m}{m! \sum_{k=0}^{m} \frac{\rho^k}{k!}}$$

  where $B(\rho, m)$ is the call blocking probability, m is the total number of channels in the trunk and $\rho = \lambda\mu$ is the total offered traffic in Erlangs (λ is the call arrival rate and μ is the mean call duration).
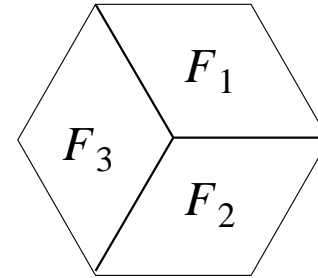
- The cell Erlang capacity accounts for the **trunking efficiency**, a phenomenon where larger groups of channels are able to carry more traffic per channel for a given blocking probability than smaller groups of channels.

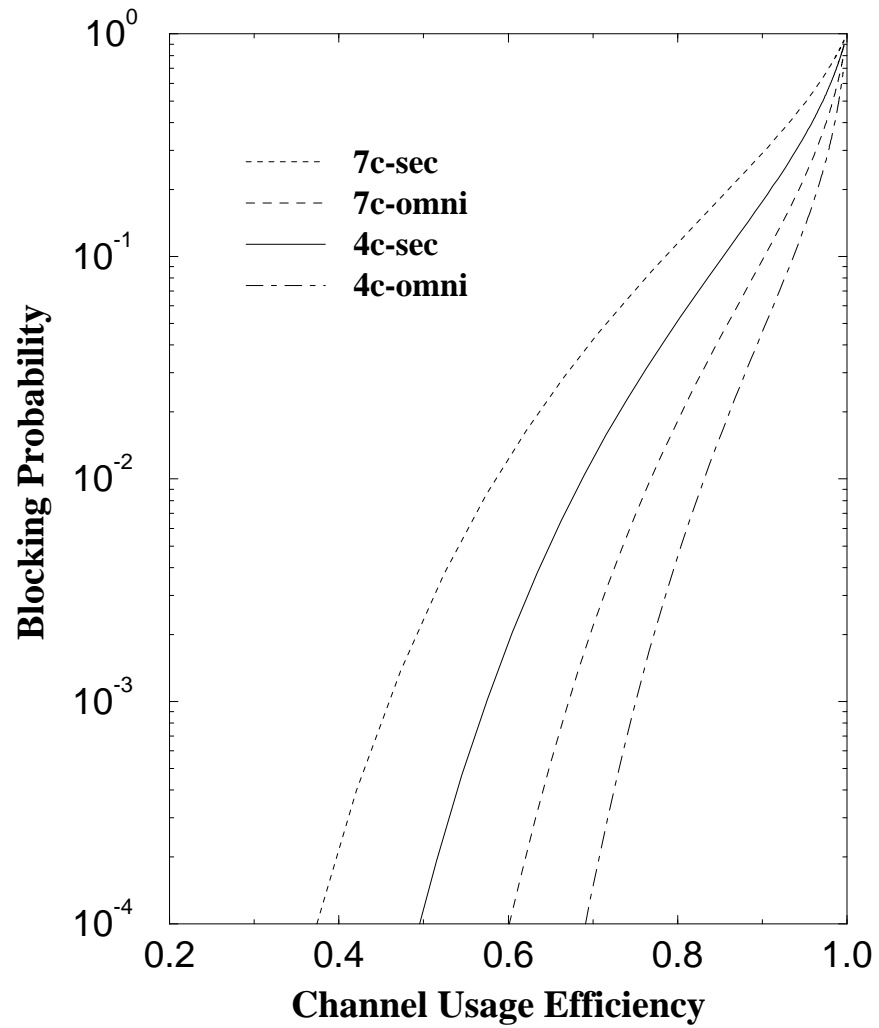Blocking probability $B(\rho, m)$ against offered traffic per channel $G_c = \rho/m$.
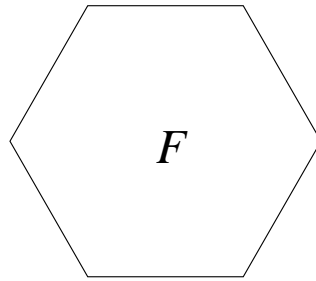
omni

3-sector

$$F = F_1 + F_2 + F_3$$

Trunkpool schemes.

Channel usage efficiency $\eta_C = \rho(1 - B(\rho, m)/m$ for different trunkpool schemes; 416 channels.

GSM Cell Capacity

- A 3/9 (3-cell/9-sector) reuse pattern is achievable for most GSM systems that employ frequency hopping; without frequency hopping, a 4/12 reuse pattern may be possible.

- GSM has 8 logical channels that are time division multiplexed onto a single radio frequency carrier, and the carriers are spaced 200 kHz apart. Therefore, the bandwidth per channel is roughly 25 kHz, which was common in first generation European analog mobile phone systems.

- In a nominal bandwidth of 1.25 MHz (uplink or downlink) there are $1250/200 = 6.25$ carriers spaced 200 kHz apart. Hence, there are $6.25/9 \approx 0.694$ carriers per sector or $6.25/3 = 2.083$ carriers/cell.

- Each carrier commonly carries half-rate traffic, such that there are 16 channels/carrier. Hence, the 3/9 reuse system has a sector capacity of 11.11 channels/sector or a cell capacity of 33.33 channels/cell in 1.25 MHz.

IS-95 Cell Capacity

- Suppose there are N users in a cell; one desired user and $N - 1$ interfering users. For the time being, ignore the interference from surrounding cells. Consider the reverse link, and assume perfectly power controlled MS transmissions that arrive chip and phase asynchronous at the BS receiver.

- Treating the co-channel signals as a Gaussian impairment, the effective carrier-to-noise ratio is (the factor of 3 accounts for chip and phase asynchronous signals)

$$\Gamma = \frac{3}{N - 1} \ ,$$

and the effective received bit energy-to-noise ratio is

$$\begin{aligned}
\frac{E_b}{N_o} &= \Gamma \times \frac{B_w}{R_b} \\
&= \frac{3G}{N - 1} \approx \frac{3G}{N} \ ,
\end{aligned}$$

where $G = B_w / R_b$.

- For a required E_b/N_o, $(E_b/N_o)_{\text{req}}$, the cell capacity is

$$N \approx \frac{3G}{(E_b/N_o)_{\text{req}}} \ .$$

IS-95 Cell Capacity (cont'd)

- Suppose that 1.25 MHz of spectrum is available and the source coder operates at $R_b = 4$ kbps. Then $G = 1250/4 = 312.5$. If $(E_b/N_o)_{\text{req}} = 6$ dB (a typical IS-95 value), then the cell capacity is roughly $N = 3 \cdot 312.5/4 \approx 234$ channels per cell. This is roughly 7 times the cell capacity of GSM.

    - This rudimentary analysis did not include out-of-cell interference which is typically 50 to 60% of the in-cell interference. This will result in a reduction of cell capacity by a factor of 1.5 and 1.6, respectively.

    - With CDMA receivers, great gains can be obtained by improving receiver sensitivity. For example, if $(E_b/N_o)_{\text{req}}$ can be reduced by 1 dB, then the cell capacity N increases by a factor of 1.26.

    - CDMA systems are known to be sensitive to power control errors. An rms power control error of 2 dB will reduce the capacity by roughly a factor of 2.

Some Elements for High Capacity

- Our emphasis is on physical wireless communications

- At the physical layer, <u>some</u> of the key elements to high capacity and good coverage in frequency reuse systems are

    - adaptive power and bandwidth efficient modulation

    - multipath-fading mitigation/exploitation (MIMO techniques for spatial multiplexing, multiuser diversity)

    - techniques to mitigate time delay spread (OFDM, equalizers, RAKE receivers)

    - co-channel interference cancellation (single and multi-antenna interference cancellation)

    - coded modulation (Turbo coding, bit interleaved coded modulation)

    - co-channel interference control (handoffs, power control, space-division multiple access)

    - coordinated multipoint or cooperative MIMO or multicell MIMO