An Efficient Algorithm for Estimating the Signal-to-Interference Ratio in TDMA Cellular Systems

Mustafa Türkboylari, Gordon L. Stüber, *Senior Member, IEEE*

Abstract—

A new algorithm is proposed for estimating the signal-to-interference ratio (SIR) in TDMA cellular systems. The SIR estimator is evaluated for use in the IS-54/136 system and a GSM-like system, and compared with existing SIR estimation algorithms. The algorithm is shown to offer comparable performance to the best known SIR estimation schemes in terms of the mean square prediction error, but with a significantly reduced computational complexity.

Keywords— **SIR estimation, signal projection.**

I. Introduction

MANY contemporary cellular radio resource management algorithms for handoffs [1], channel assignment [2], [3], and power control [4] assume fast and accurate measurements of the signal-to-interference plus noise ratio, S/(I+N). Several methods have been recently developed to generate real time estimates of the S/(I+N) in TDMA cellular systems. One method, called *Interference Projection* (IP) uses the training and/or color code sequences that are typically present within cellular TDMA slots to obtain an unbiased estimate of S/(I+N) [5]. Another method uses the autocorrelation sequence of the received signal samples over a short time scale [6]. Still another method, called *Subspace-based* (SB), estimates the S/(I+N) by from the eigenvalues of the covariance matrix of the received signal sequence [7].

This paper develops another efficient S/(I+N) estimation technique, called *Signal Projection* (SP) for TDMA cellular systems. The SP method has a computational complexity comparable to the IP method and an average absolute S/(I+N) prediction error comparable to the SB method. Section II presents the system and channel model, and the proposed S/(I+N) estimation technique. Section III contains numerical results and discussion, followed by concluding remarks in Section IV.

II. Discrete-Time Channel Model

Following [5], the overall system consisting of the transmit filter, channel, matched filter, and baud-rate sampler is modeled as an M-tap, T-spaced, transversal filter, where T is the baud duration. The filter taps are described by the vector $\mathbf{f} = [f_1, \ldots, f_M]^T$ and the filter outputs are $y_n = \sum_{k=1}^{M} a_{n-k} f_k + w_n$, where $\{a_n\}$ is the information sequence and the $\{w_n\}$ is the sample sequence of interference plus noise. If $\mathbf{y} \triangleq [y_1, \ldots, y_L]^T$, then $\mathbf{y} = \mathbf{A}\mathbf{f} + \mathbf{w}$, where $\mathbf{A}$ is an $L \times M$ Toeplitz *convolution* matrix and $\mathbf{w} = [w_1, \ldots, w_L]^T$. If $\boldsymbol{\alpha} = [\alpha_1 \ldots \alpha_{L+M-1}]$ is the length-$(L+M-1)$ *training sequence* embedded in each TDMA time slot, then

$$\mathbf{A} = \begin{bmatrix} \alpha_M & \alpha_{M-1} & \cdots & \alpha_1 \\ \alpha_{M+1} & \alpha_M & \cdots & \alpha_2 \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{M+L-1} & \alpha_{M+L-2} & \cdots & \alpha_L \end{bmatrix}. \quad (1)$$

Note that the matrix $\mathbf{A}$ could also be formed by using decisions on the transmitted symbols [5]. Assuming N_I co-channel interferers, $\mathbf{w} = \sum_{k=1}^{N_I} \mathbf{A}_k \mathbf{f}_k + \mathbf{n}$, where $\mathbf{n}$ is a zero-mean, discrete-time, white Gaussian process. For asynchronous co-channel signals (e.g., IS-54/136, PDC, and GSM), the received co-channel signals are uncorrelated and mutually uncorrelated, implying that $\mathbf{w}$ and $\mathbf{A}$ are mutually uncorrelated. Assuming a zero-mean signal constellation, $\mathbf{y}$ and $\mathbf{w}$ have the property $\mathrm{E}[y_j y_k^*] = 0$ and $\mathrm{E}[w_j w_k^*] = 0$ for $j \neq k$.

A. Estimating S/(I+N)

Suppose that $L > M$. Let $\mathcal{R}(\mathbf{A})$ and $\mathcal{N}(\mathbf{A}^T)$ denote the *range space* and *left null space* of $\mathbf{A}$, respectively. Also, let $Q = \dim(\mathcal{R}(\mathbf{A}))$ and $P = \dim(\mathcal{N}(\mathbf{A}^T))$. Let $\{\mathbf{e}_1, \ldots, \mathbf{e}_P\}$ be the orthonormal basis for $\mathcal{N}(\mathbf{A}^T)$ and $\{\mathbf{e}_{P+1}, \ldots, \mathbf{e}_L\}$ be the orthonormal basis for $\mathcal{R}(\mathbf{A})$. Let $\mathbf{x}$ be the orthogonal projection of $\mathbf{w}$ onto $\mathcal{N}(\mathbf{A}^T)$. Consider

$$\mathrm{E}[||\mathbf{x}||^2] = \mathrm{E}[\sum_{i=1}^{P} |\langle \mathbf{e}_i, \mathbf{w}\rangle|^2] = \sum_{i=1}^{P} \mathrm{E}[|\langle \mathbf{e}_i, \mathbf{w}\rangle|^2], \quad (2)$$

where $\langle \mathbf{x}, \mathbf{w}\rangle = \mathbf{x}^H \mathbf{w}$ is the inner product and $||\mathbf{x}||^2 = \langle \mathbf{x}, \mathbf{x}\rangle$. However,

$$\begin{aligned} \mathrm{E}[|\langle \mathbf{e}_i, \mathbf{w}\rangle|^2] &= \sum_{j=1}^{L}\sum_{k=1}^{L} e_{i,j} e_{i,k}^* \mathrm{E}[w_j w_k^*] \\ &= \sum_{j=1}^{L} |e_{i,j}|^2 \sigma_{I+N}^2 = \sigma_{I+N}^2 \ . \end{aligned} \quad (3)$$

This research was supported by the National Science Foundation under Grant NCR-9523969. This paper was presented in part at 1997 Global Telecommunications Conference, Communications Theory Mini-conference, Phoenix, AZ, November 5-7, 1997.

M. Türkboylari and G. L. Stüber are with the School of Electrical and Computer Engineering, Georgia Institute of Technology, Atlanta, GA 30332-0250 USA.

By combining (2) and (3), $\sigma^2_{I+N} = \frac{1}{P}\mathrm{E}[\mathbf{x}^H\mathbf{x}]$, leading to the unbiased estimate

$$\hat{\sigma}^2_{I+N} = \frac{1}{P}||\mathbf{x}||^2 = \frac{1}{P}\sum_{i=1}^{P}|\langle \mathbf{e}_i, \mathbf{w}\rangle|^2 \ . \tag{4}$$

Since $\mathbf{e}_i \in \mathcal{N}(\mathbf{A}^T),\ i = 1, \ldots, P$

$$\langle \mathbf{e}_i, \mathbf{y}\rangle = \langle \mathbf{e}_i, \mathbf{Af} + \mathbf{w}\rangle = \mathbf{e}_i^H\mathbf{Af} + \mathbf{e}_i^H\mathbf{w} = \mathbf{e}_i^H\mathbf{w} = \langle \mathbf{e}_i, \mathbf{w}\rangle \ . \tag{5}$$

Thus, (4) becomes

$$\hat{\sigma}_{I+N} = \frac{1}{P}\sum_{i=1}^{P}|\langle \mathbf{e}_i, \mathbf{y}\rangle|^2 = \frac{1}{P}(||\mathbf{y}||^2 - \sum_{i=P+1}^{L}|\langle \mathbf{e}_i, \mathbf{y}\rangle|^2), \tag{6}$$

where the last equality follows from *Pythagorean Theorem* since $\mathcal{N}(\mathbf{A}^T) = \mathcal{R}(\mathbf{A})^\perp$.In [5], the estimate $\hat{\sigma}^2_{I+N}$ is generated by projecting $\mathbf{y}$ onto a single vector, say $\mathbf{c}$, in $\mathcal{N}(\mathbf{A}^T)$; however, the approach here projects $\mathbf{y}$ onto the entire signal subspace $\mathcal{R}(\mathbf{A})$.

The received S/(I+N) can be estimated by separating the signal power from the interference plus noise power. The signal plus interference plus noise power is

$$\sigma^2_{S+I+N} = \frac{1}{L}\mathrm{E}[\mathbf{y}^H\mathbf{y}] = \sigma^2_S + \sigma^2_{I+N}, \tag{7}$$

Hence, S/(I+N) can be estimated as

$$\mathrm{S/(I+N)_{est}} = \frac{\hat{\sigma}^2_S}{\hat{\sigma}^2_{I+N}} = \left(\frac{\hat{\sigma}^2_{S+I+N}}{\hat{\sigma}^2_{I+N}} - 1\right) \ . \tag{8}$$

Combining (8) and (6) gives

$$\mathrm{S/(I+N)_{est}} = \frac{\frac{1}{L}\mathbf{y}^H\mathbf{y}}{\frac{1}{P}(||\mathbf{y}||^2 - \sum_{i=P+1}^{L}|\langle \mathbf{e}_i, \mathbf{y}\rangle|^2)} - 1 \ . \tag{9}$$

If S/(I+N) is estimated from N different received $\mathbf{y}$, then (9) has the more general form

$$\mathrm{S/(I+N)_{est}} = \frac{\sum_{i=1}^{N}||\mathbf{y}_i||^2}{\sum_{i=1}^{N}\frac{L}{P_i}(||\mathbf{y}_i||^2 - \sum_{k=P_i+1}^{L}|\langle \mathbf{e}^i_k, \mathbf{y}_i\rangle|^2)} - 1 \ . \tag{10}$$

Since the S/(I+N) estimator in (10) projects the received signal vectors onto the signal subspace, the proposed algorithm is called the *Signal Projection* estimator.

The columns of $\mathbf{A}$ are readily available as a basis for $\mathcal{R}(\mathbf{A})$. They just need to be orthonormalized by using the *Gram-Schmidt* orthonormalization procedure, requiring M^2L multiplications and $\frac{1}{2}M(M+1)L$ additions. For situations where the matrix $\mathbf{A}$ is derived from a training sequence, the basis vectors $\{\mathbf{e}_{P+1}, \ldots, \mathbf{e}_L\}$ can be precomputed and stored at the receiver. Notice that the estimate in (9) improves with increasing P[1], and the computational load in computing the basis vectors diminishes with increasing P, i.e., when L is chosen as large as possible relative to M.

[1] P is ultimately limited by the length of the training sequence, $L + M - 1$.

Fig. 1. Mean SIR estimation error against the averaging time for the IS-54/136 system (N_I=6).

Finally, a value for M must be chosen in the SP estimator when forming the $\mathbf{A}$ matrix. To choose an appropriate M, we note that a typical radio receiver employs an equalizer that is designed for some particular value of M; sometimes the channel will be underequalized or overequalized. This same value of M could be used to form the $\mathbf{A}$ matrix. Such an approach has the practical benefit of tuning the S/(I+N) estimate to what the equalizer actually sees.

III. Numerical Results and Discussion

The performance of the SP estimator was evaluated by software simulation for both an IS-54/136 system [8] and a GSM-like system[2]. Only the known training and color code symbols were used to derive the estimate. For IS-54/136, we assume full-rate operation (two bursts per frame), where the 14-symbol training sequence and 6-symbol colour code sequence in each burst are used to form the S/(I+N) estimate. For GSM, the 26-symbol training sequence in each burst is used to form the S/(I+N) estimate.

Our simulations assume a two-equal-ray, baud-spaced, Rayleigh fading channel with uncorrelated taps for both the IS-54/136 and GSM-like systems. It is not important to use the same channel model for both systems, because we are not trying to make comparisons between systems, but rather compare the relative performance of the different S/(I+N) estimators when they are applied to each system. Additive white Gaussian noise at 20 dB below the interference power was also included. In all simulations the true S/I is 5 dB and there are six equal power co-channel interferers.

1) IS-54/136 System: Two cases are examined; one uses the training sequences only and the other uses both the training and colour code sequences. Fig. 1 plots the mean absolute SIR estimation error $\mathrm{E}[|\mathrm{S/(I+N)}_{(\mathrm{dB})} -$

[2] Our GSM-like system is similar to GSM, except that GMSK is replaced with BPSK.

Fig. 2. Error variance against the averaging time for the IS-54/136 system (N_I=6).

Fig. 3. Mean SIR estimation error against the averaging time for the IS-54/136 system (N_I=6).

Fig. 4. Mean SIR error against the averaging time for a GSM-like system (N_I=6).

Fig. 5. Error variance against the averaging time for a GSM-like system (N_I=6).

$\mathrm{S/(I{+}N)}_{\mathrm{est(dB)}}|]$[3] against the averaging interval for the SP, IP and, SB estimators. Only the 14-symbol training sequence is used. The IP estimator performs poorly compared to the SP and SB estimators for all MS speeds. The SB estimator has a large initial error, but converges and offers comparable performance to the SP estimator. After one second, the SP estimator yields average errors of 0.5 dB at 100 km/h and 1 dB at 5 km/h.

Although not shown, all the S/(I+N) estimators are insensitive to the true S/(I+N). Fig. 2 shows the error variances $\mathrm{E}[|\mathrm{S/(I{+}N)}_{\mathrm{(dB)}} - \mathrm{S/(I{+}N)}_{\mathrm{est(dB)}}|^2]$. Finally, Fig. 3 shows the results when the SP estimator uses both the training and colour code sequences. Using the colour code sequences does not provide any significant improvement because, in this case, $L = 5$ and $P = 3$ which are not large enough to improve the S/(I+N) estimate.

[3] All the quantities are converted to dB before any averaging is done in all the figures.

2) GSM-like System: Fig. 4 shows the mean absolute SIR estimation error against the averaging interval for the SP and SB estimators. Once again, the SP and SB estimators have comparable performance, while the SB estimator has a large initial error. After one second, the S/(I+N) can be estimated to within an average error of about 0.3 dB at 100 km/h and 1 dB at 5 km/h. Fig. 5 shows the corresponding error variance.

A. Computational Issues

The SB and SP estimators have comparable performance. However, SP has better computational efficiency. For a given matrix $\mathbf{A}$, the SP estimator requires M^2L multiplications and $\frac{1}{2}M(M+1)L$ additions obtain the orthonormal basis for $\mathcal{R}(\mathbf{A})$. On the other hand, for each S/(I+N) estimate the SB estimator requires the eigenvalues of an $L \times L$ Hermitian covariance matrix $\mathbf{B}$, L different values of a sphericity test function, and a search algorithm for a vector of length L. Finding the eigenvalues of a matrix is computationally intensive. The QR algorithm is the most efficient and widely used method to calculate the eigenvalues of a matrix [9]. The QR algorithm factors the matrix $\mathbf{B}$ as $\mathbf{B} = \mathbf{QR}$ where $\mathbf{R}$ upper triangular and $\mathbf{Q}$ orthogonal. Each iteration of the QR algorithm requires $O(L^3)$ multiplications and additions. The number of iterations of in the QR method depends on the eigenvalues of the matrix $\mathbf{B}$. If the eigenvalues, $\{\lambda_i\}$, are listed in order of descending magnitude, i.e., $|\lambda_1| > |\lambda_2| \cdots > |\lambda_L| > 0$, then convergence is rapid only if the quantity $\max_i |\lambda_{i+1}/\lambda_i|$ is small [9]. In a typical application $M = 2$ and $Q \leq M$, and $L = 13$. From [7], $\max_i |\lambda_{i+1}/\lambda_i| = 1$, implying a slow convergence rate[4]. For these parameters our simulations required an average of 106000 flops to obtain the eigenvalues for each sample covariance matrix.

IV. Concluding Remarks

A computationally simple and accurate *signal projection* S/(I+N) estimation technique was developed and applied to TDMA cellular systems. The new estimation technique was shown offer comparable performance to the *subspace-based* S/(I+N) estimation technique, but with a significantly reduced computational complexity.

References

[1] C. N. Chuah, R. D. Yates, and D. J. Goodman, "Integrated Dynamic Radio Resource Management," *IEEE Veh. Tech. Conf.*, pp. 584–588, Jul. 1995.

[2] R. W. Nettleton and G. R. Schoemer, "A High Capacity Assignment Method for Cellular Mobile Telephone Systems," *IEEE Veh. Tech. Conf.*, pp. 359–367, 1989.

[3] D. J. Goodman, S. A. Grandhi, and R. Vijayan, "Distributed Dynamic Channel Assignment Schemes," *IEEE Veh. Tech. Conf.*, pp. 532–535, May 1993.

[4] S. Ariyavisitakul and L. F. Chang, "Signal and Interference Statistics of a CDMA System with Feedback Power Control," *IEEE Trans. Commun.*, Vol. 41, pp. 1626–1634, Nov. 1993.

[5] M. D. Austin and G. L. Stüber, "In-Service Signal Quality Estimation for TDMA Cellular Systems," *Kluwer Wireless Personal Commun.*, Vol. 2, pp. 245–254, 1995.

[6] A. Brandão, L. Lopes, and D. McLernon, "Quality Assessment for Pre-Detection Diversity Switching," *IEEE PIMRC*, pp. 577–581, Sep. 1995.

[7] M. Andersin, N. B. Mandayam, and R. D. Yates, "Subspace Based Estimation of the Signal to Interference Ratio for TDMA Cellular Systems," in *IEEE Veh. Tech. Conf.*, pp. 1155–1159, Apr. 1996.

[8] *EIA/TIA Interim Standard, Cellular System Dual Mode Mobile Station-Base Station Compatibility Standard*, Jan. 1990.

[9] K. E. Atkinson, *An Introduction to Numerical Analysis.* John Wiley & Sons, 1989.

[4] Since a sample covariance matrix is used rather than the true covariance matrix, $\max_i |\lambda_{i+1}/\lambda_i|$ is not 1 but close to 1.